KEYWORDS

Accuracy	**Kilometre**
Area	**Measurement**
Capacity	**Metre**
Centimetre	**Millimetre**
Composite figures	**Perimeter**
Hectare	**Prism**
	Volume

Many questions in this chapter use fractions and decimals. Students can review Chapters 2 and 3 for assistance.

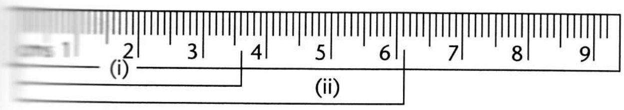
Units of Length

In Australia, the basic unit of length is the metre (abbreviated m). Other lengths are multiples of the metre, or divisions of it:

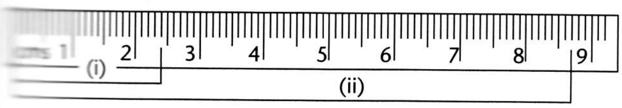
1000 mm = 1 m
100 cm = 1 m
$\therefore$ 10 mm = 1 cm
Also 1000 m = 1 km

For Example

1 **a** Write down the length of each interval to the nearest millimetre:



b Write down the length of each interval to the nearest centimetre:



2 **a** Convert the following to the given units:

- **i** 4 m = ____ cm
- **ii** 3 km = ____ m
- **iii** 7000 mm = ____ m
- **iv** 270 cm = ____ m
- **v** 64 mm = ____ cm
- **vi** 3 400 000 mm = ____ km
- **vii** 800 mm = ____ m
- **viii** 0.35 km = ____ m

b Complete the following (using sensible units):

- **i** Length of pen = 15 ____
- **ii** Height of door = 2 ____
- **iii** Distance from Singleton to Walgett = 511 ____
- **iv** Length of a fingernail = 11 ___

c If a 20c coin rolls 85 mm in one revolution:

- **i** How far, in metres, will it travel in 25 revolutions?
- **ii** How many revolutions of the coin is required for it to travel 30.6 m?

1 **a** **i** 36 mm **ii** 61 mm

b **i** 2 cm **ii** 9 cm

2 **a**

- **i** $4 \times 100 = 400$ i.e. 400 cm
- **ii** $3 \times 1000 = 3000$ i.e. 3000 m
- **iii** $7000 \div 1000 = 7$ i.e. 7 m
- **iv** $270 \div 100 = 2.7$ i.e. 2.7 m
- **v** $64 \div 10 = 6.4$ i.e. 6.4 cm
- **vi** $3\,400\,000 \div 1\,000\,000 = 3.4$ i.e. 3.4 km

	vii	$800 \div 1000 = 0.8$	i.e. 0.8 m	
	viii	$0.35 \times 1000 = 350$	i.e. 350 m	
b	i	15 cm	ii	2 m
	iii	511 km	iv	11 mm

c i $85 \text{ mm} = 0.085 \text{ m}$

$\therefore$ Distance $= 0.085 \text{ m} \times 25$

$= 2.125 \text{ m}$

ii Each revolution $= 0.085$ m

$\therefore$ No. of revolutions

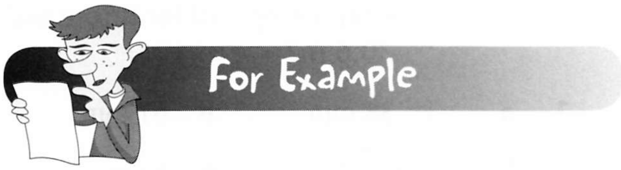
$= 30.6 \div 0.085$

$= 360$

$\therefore$ 360 revolutions are needed.

Limits of Measurement

A variety of measuring instruments are used, depending on the length of the object to be measured; for example, a ruler, tape measure or trundle wheel. However, when we measure we are really only approximating. If we say a length is 8 cm, the exact measure could be between 7.5 cm and 8.5 cm. This is called the **limit of measurement**.



Find the limit of measurement of the following:

1 A length of hair is measured as 17 cm.

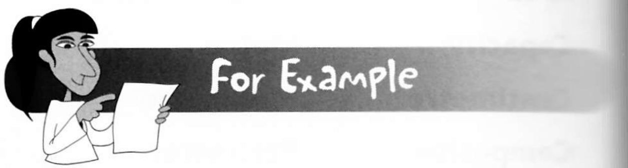
2 A piece of timber is measured as 4.2 m.

1 16.5 cm to 17.5 cm

2 4.15 m to 4.25 m

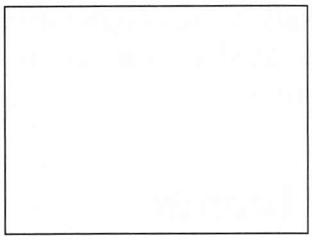
Perimeter

The perimeter of a shape is the distance around the shape.

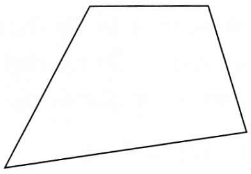


1 Measure the perimeter of these shapes with your ruler, leaving your answer in centimetres (cm):

a

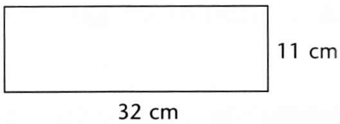


b



2 Find the perimeter of the following:

a



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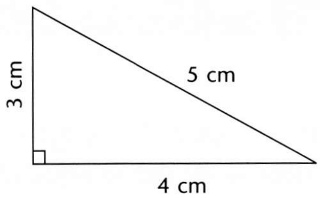
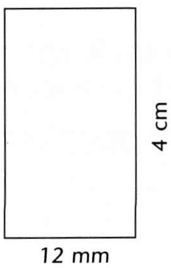
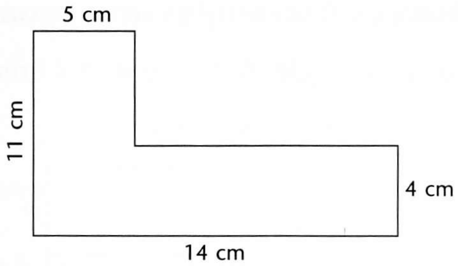


c A regular hexagon with side lengths of 14 cm.

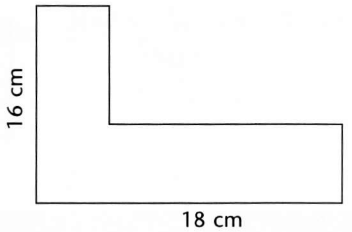
d



e



f



3 **a** If the perimeter of a square was 116 cm, find the length of each side.

b A farmer wishes to fence a paddock that measures 125 metres by 220 metres:

i Find the perimeter of the paddock.

ii How much will it cost the farmer at 95 cents per metre to fence the paddock?

1 **a** $P = 4 + 3 + 4 + 3$

$= 14$ $\therefore$ Perimeter is 14 cm.

b $P = 2.4 + 1.6 + 1.7 + 3.3$

$= 9.0$ $\therefore$ Perimeter is 9.0 cm.

2 **a** $P = 32 + 11 + 32 + 11$

$= 2(32 + 11)$

$= 2(43)$

$= 86$ $\therefore$ Perimeter is 86 cm.

b $P = 3 + 4 + 5$

$= 12$ $\therefore$ Perimeter is 12 cm.

c $P = 6 \times 14$ [A hexagon has 6 sides.]

$= 84$

$\therefore$ Perimeter is 84 cm.

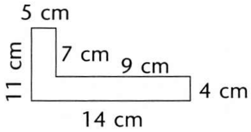
d $P = 12 + 40 + 12 + 40$ [4 cm = 40 mm]

$= 2(12 + 40)$

$= 2(52)$

$= 104$ $\therefore$ Perimeter is 104 mm.

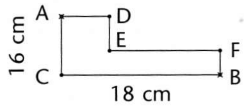
e



$P = 14 + 4 + 9 + 7 + 5 + 11$

$= 50$ $\therefore$ Perimeter is 50 cm.

f



From A to B via C is the same distance as A to B via D, E, F:

$P = 2(16 + 18)$

$= 2(34)$

$= 68$ $\therefore$ Perimeter is 68 cm.

3 **a** Perimeter = 116 cm

$\therefore$ Side $= 116 \div 4$

$= 29$

$\therefore$ Length of side is 29 cm.

b **i** $P = 2(125 + 220)$

$= 2(345)$

$= 690$ $\therefore$ Perimeter is 690 m.

ii Cost = $690 \times \$0.95$

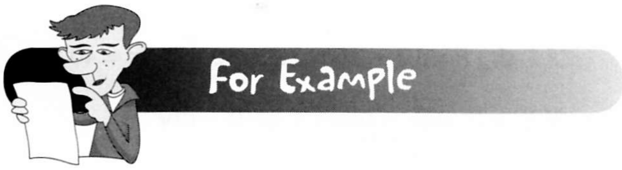
[Change 95c to \$0.95]

$= \$655.50$

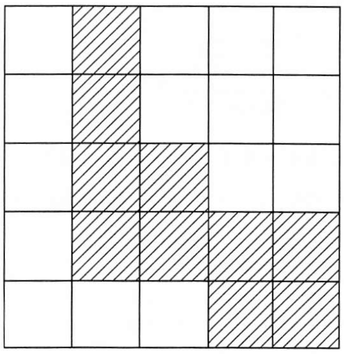
$\therefore$ Cost to farmer is \$655.50.

Area

Area is a measure of the space contained within a plane shape.

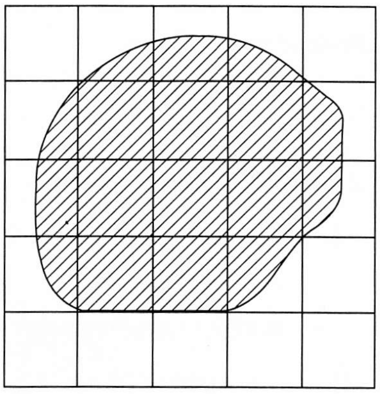


1 Find the area of the shaded region:



Each square on the diagram measures 1 unit by 1 unit, called a square unit (i.e. $unit^2$).

2 Find the approximate area of the shaded region:



1 By counting, Area = 10 square units

$= 10 \text{ units}^2$

2 By counting, Area = 8 squares plus 9 'half squares'

$= 8 + 4\frac{1}{2}$

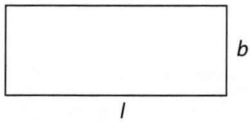
$= 12\frac{1}{2}$

$\therefore$ Area is approximately $12\frac{1}{2}$ units^2.

Area of Rectangles and Squares

For a rectangle, Area = length $\times$ breadth

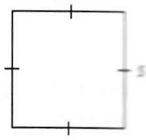
i.e. $A = lb$

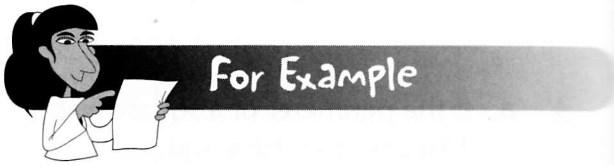


For a square, Area = side $\times$ side

$= \text{side}^2$

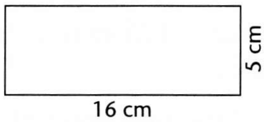
$\therefore A = s^2$



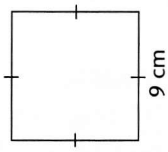


Find the area:

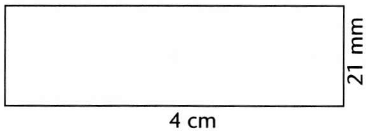
1



2



3



Remember that if the units in the question are in cm, then the area units are cm^2.

1 $A = lb$

$= 16 \times 5$

$= 80$

$\therefore$ Area is 80 cm^2.

2 $A = s^2$

$= 9^2$

$= 81$

$\therefore$ Area is 81 cm^2.

3 4 cm = 40 mm

$\therefore A = lb$

$= 40 \times 21$

$= 840$

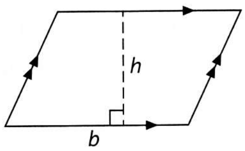
$\therefore$ Area is 840 mm^2.

[A good way to learn a formula is to write it down in your solution before you use it.]

Area of a Parallelogram

For a parallelogram, Area = base × height

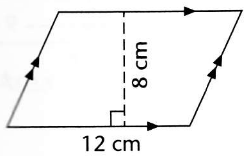
i.e. $A = bh$



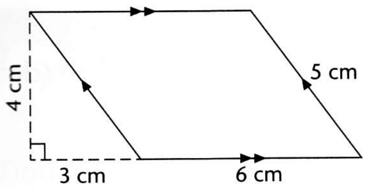
For Example

Find the area:

1



2



1 $A = bh$

$= 12 \times 8$

$= 96$

$\therefore$ Area is 96 cm^2.

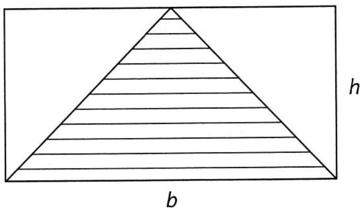
2 $A = bh$

$= 6 \times 4$

$= 24$

$\therefore$ Area is 24 cm^2.

Area of a Triangle



The area of the shaded triangle is **exactly** half the area of the rectangle.

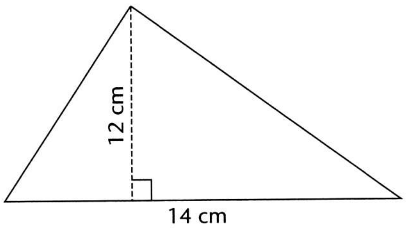
$\therefore$ For a triangle, Area = $\frac{1}{2}$ × base × height

i.e. $A = \frac{1}{2}bh$ [$A = \frac{1}{2} \times b \times h$]

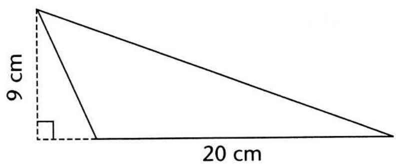
For Example

Find the area:

1



2



1 $A = \frac{1}{2}bh$

$= \frac{1}{2} \times 14 \times 12$

$= 7 \times 12$

$= 84$

$\therefore$ Area is 84 cm^2.

2 $A = \frac{1}{2}bh$

$= \frac{1}{2} \times 20 \times 9$

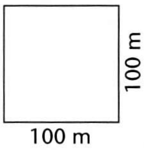
$= 10 \times 9$

$= 90$

$\therefore$ Area is 90 cm^2.

Units Used in Area Measurement

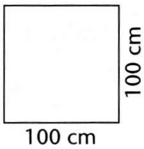
- One hectare (ha) is a square area 100 metres long by 100 metres wide:



$$1 \text{ hectare} = 100 \times 100 \text{ m}^2 = 10\,000 \text{ m}^2$$

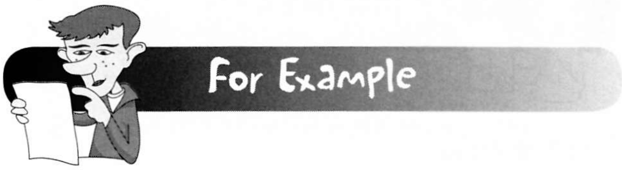
[1 hectare = 10 000 m^2]

- Also:



$$1 \text{ m}^2 = 100 \times 100 \text{ cm}^2 = 10\,000 \text{ cm}^2$$

[1 m^2 = 10 000 cm^2]



Complete these statements:

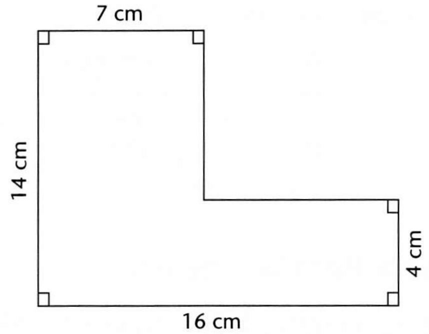
1. 30 000 cm^2 = ____ m^2
2. 4.2 m^2 = ____ cm^2
3. 2 ha = ____ m^2
4. 84 000 m^2 = ____ ha

1. $30\,000 \div 10\,000 = 3 \text{ m}^2$
2. $4.2 \times 10\,000 = 42\,000 \text{ cm}^2$
3. $2 \times 10\,000 = 20\,000 \text{ m}^2$
4. $84\,000 \div 10\,000 = 8.4 \text{ ha}$

Composite Areas and Other Area Problems

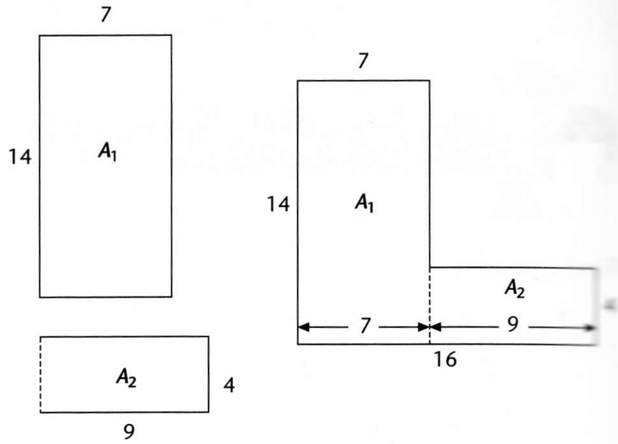
Sometimes the area to be calculated is a combination of two or more regular shapes; for example, two rectangles, or a rectangle and a triangle.

Find the area of this composite shape (all angles are right angles):



Method 1

Divide the shape into two rectangles with a vertical (or horizontal) line. Call the areas formed A_1 and A_2:



$\therefore$ Total area = $A_1 + A_2$

For A_1 we have used a vertical line (dotted):

$$A_1 = lb = 14 \times 7 = 98$$

For A_2, we must find the length before we can calculate the area. This is 16 – 7 = 9 cm:

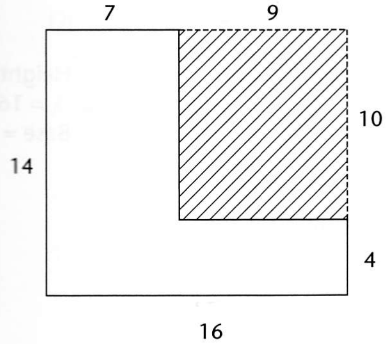
$$A_2 = lb = 9 \times 4 = 36$$

Total area = 98 + 36
= 134 (i.e. $A_1 + A_2$)

$\therefore$ Area is 134 cm^2.

Method 2

This method involves completing the full rectangle (dotted line) and then subtracting the extra piece (shaded):



Area large rectangle = lb
$= 16 \times 14$
$= 224$

Area shaded = lb
$= 9 \times 10$
$= 90$

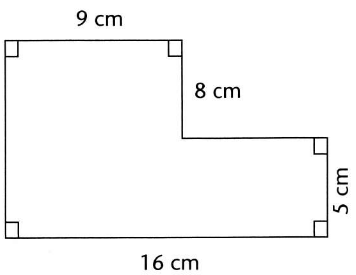
Required area = $224 - 90$
$= 134$

$\therefore$ Area is 134 cm^2.

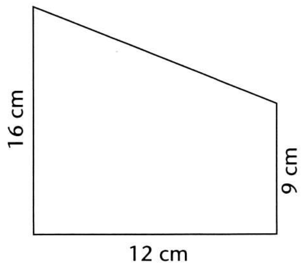
For Example

1 Find the areas:

a

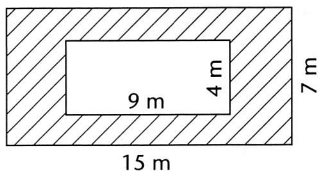


b

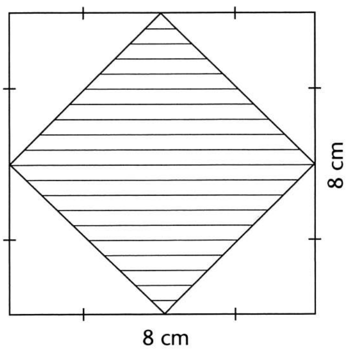


2 Find the area of the shaded regions:

a



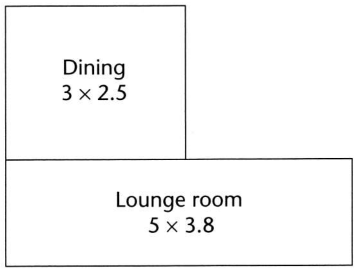
b



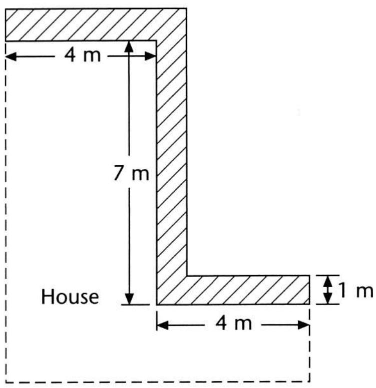
3 The diagram below shows part of the plan of a house where the size of rooms is expressed in metres. Find the:

a Area of each room

b Total cost of carpeting the lounge and dining areas if carpet costs $30/$m^2$.



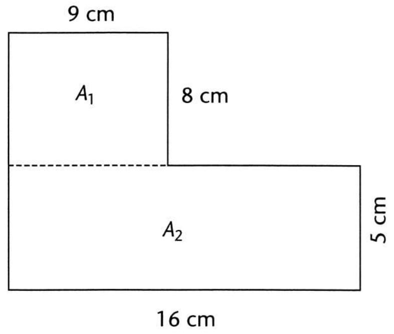
4 Kevin is to lay a 1 m wide footpath around the back of his house using rectangular pavers that are 20 cm by 10 cm:



a Find the area to be paved.

b How many pavers will Kevin need?

1 **a**



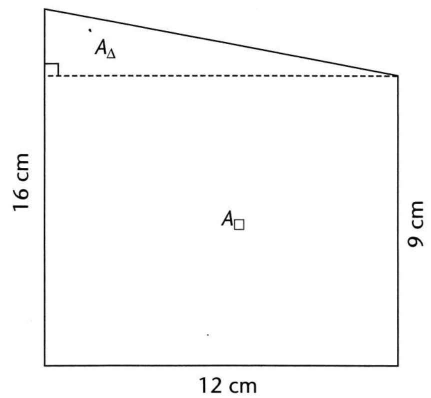
$A_1 = lb$
$= 9 \times 8$
$= 72$

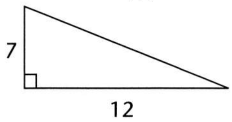
$A_2 = lb$
$= 16 \times 5$
$= 80$

Total area $= A_1 + A_2$
$= 72 + 80$
$= 152$

Area is 152 cm^2.

b





This time, with a horizontal line (dotted) we form a triangle and a rectangle.

$\text{Area}_\square = lb$
$= 12 \times 9$
$= 108$

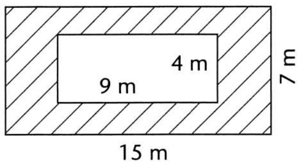
$\text{Area}_\triangle = \frac{1}{2}bh$
$= \frac{1}{2} \times 12 \times 7$
$= 42$

[Height of $\triangle = 16 - 9 = 7$
Base = 12]

Total area $= A_\square + A_\triangle$
$= 108 + 42$
$= 150$

Area is 150 cm^2.

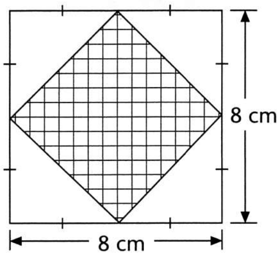
2 **a**



Shaded area = Area of large rectangle − Area of small rectangle
$= 15 \times 7 - 9 \times 4$
$= 105 - 36$ [$A = lb$]
$= 69$

$\therefore$ Shaded area is 69 m^2.

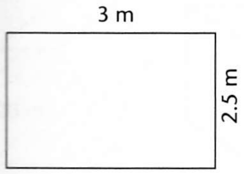
b



Shaded area = Area of square − Area of 4 triangles
$= 8 \times 8 - 4(\frac{1}{2} \times 4 \times 4)$
$= 64 - 4(8)$
$= 64 - 32$
$= 32$

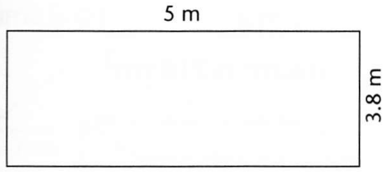
$\therefore$ Shaded area is 32 cm^2.

3 **a**



Dining room: $A = 3 \times 2.5$
$= 7.5$

$\therefore$ Area is 7.5 m^2.



Lounge room: $A = 5 \times 3.8$
$= 19\ m^2$

$\therefore$ Area is 19 m^2.

b Total area $= 7.5 + 19$
$= 26.5\ m^2$

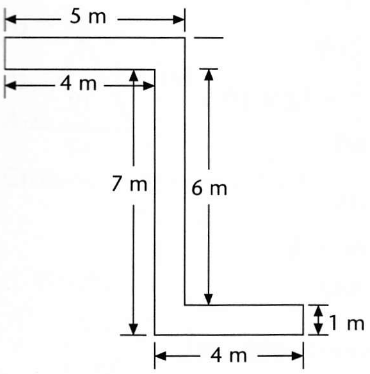
Total cost $= 26.5 \times \$30$
$= \$795$

$\therefore$ Total cost is \$795.

4 **a** Total area $= 5 \times 1 + 6 \times 1 + 4 \times 1$
$= 15$ [$A = lb$]

Total area is 15 m^2

Area to be paved is 15 m^2.



b Area of each paver $= 20\ cm \times 10\ cm$
$= 200\ cm^2$

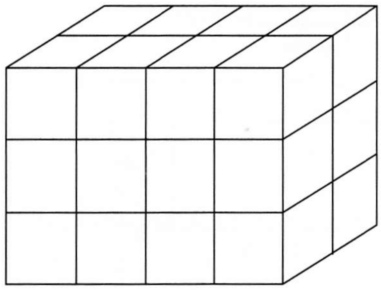
Now, total area in $cm^2 = 15 \times 10000$
$= 150000\ cm^2$

$\therefore$ No. of pavers $= 150000 \div 200$
$= 750$

$\therefore$ 750 pavers are required.

Volume

Volume is a measure of the space contained in a solid shape:

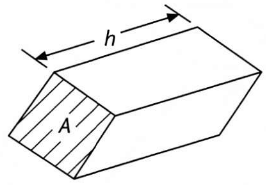


[Each cube measures 1 cm by 1 cm by 1 cm, which is called a cubic centimetre (i.e. cm^3).]

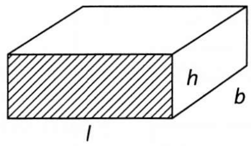
By 'counting' we can see that the volume of this solid is $4 \times 2 \times 3 = 24\ cm^3$.

Volumes of Prisms

$V = Ah$, where V = Volume
A = Area of cross section
h = height



For a rectangular prism, this can be thought of as:



Volume = length × breadth × height

[as Area = lb]

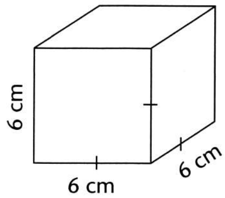
$V = lbh$

[Remember units of volume are $units^3$.]

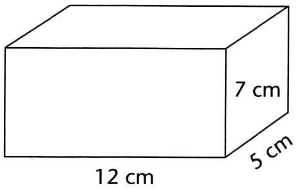
For Example

Find the volume:

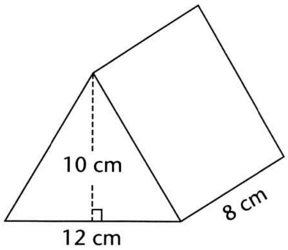
1



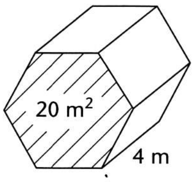
2



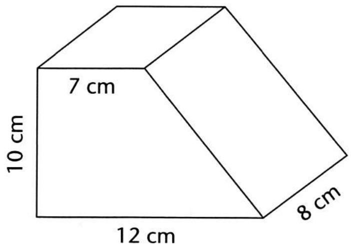
3



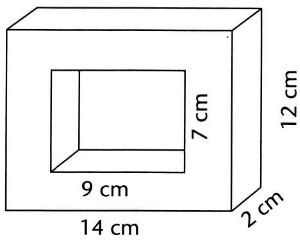
4



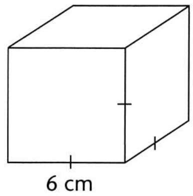
5



6



1



Cube (type of rectangular prism)

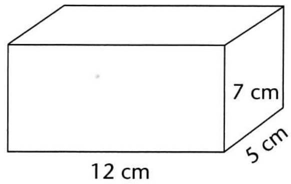
$V = lbh$

$\therefore\ V = 6 \times 6 \times 6$

$= 216$

$\therefore$ Volume is 216 cm^3.

2



(Rectangular prism)

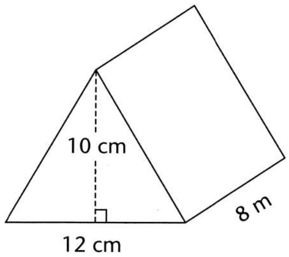
$V = lbh$

$\therefore\ V = 12 \times 7 \times 5$

$= 420$

$\therefore$ Volume is 420 cm^3.

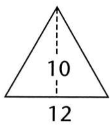
3



$A = \frac{1}{2}bh$

$= \frac{1}{2} \times 12 \times 10$

$= 60$



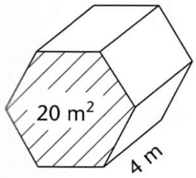
$V = Ah$

$= 60 \times 8$

$= 480$

Volume is 480 cm^3.

4



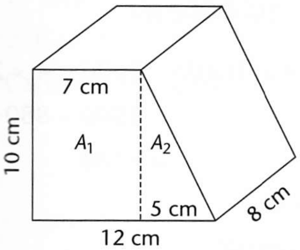
$V = Ah$

$= 20 \times 4$

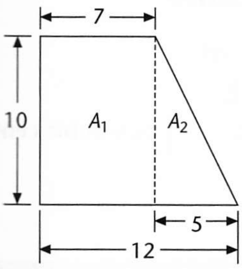
$= 80$

Volume is 80 m^3.

5



For area of cross-section:



$A_1 = lb$

$A_1 = 10 \times 7$

$A_1 = 70$

$A_2 = \frac{1}{2}bh$

$A_2 = \frac{1}{2} \times 5 \times 10$

$A_2 = 25$

Cross-section $A = 70 + 25$

$= 95$

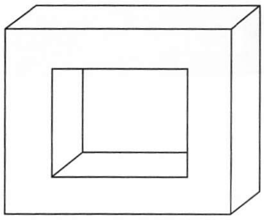
$\therefore V = Ah$

$= 95 \times 8$

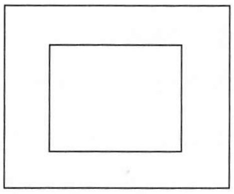
$= 760$

$\therefore$ Volume is 760 cm^3.

6



Area of cross section:



A = Area of large rect. – Area of small rect.

$= 14 \times 12 - 9 \times 7$

$= 168 - 63$

$= 105$

$\therefore V = Ah$

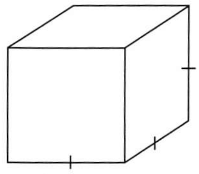
$= 105 \times 2$

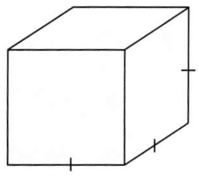
$= 210$

Volume is 210 cm^3.

Units Used in Volume Measurement

Here we have two identical cubes, one with dimensions in cm, the other in mm:





$V = 1 \times 1 \times 1$

$= 1$

Volume is 1 cm^3

$V = 10 \times 10 \times 10$

$= 1000$

Volume is 1000 mm^3

$\therefore 1\ cm^3 = 1000\ mm^3$

Complete:

1 $5\ cm^3 = \_\_\_\_\ mm^3$

2 $25\,000\ mm^3 = \_\_\_\_\ cm^3$

1 $5 \times 1000 = 5000$

$\therefore 5\ cm^3 = 5000\ mm^3$

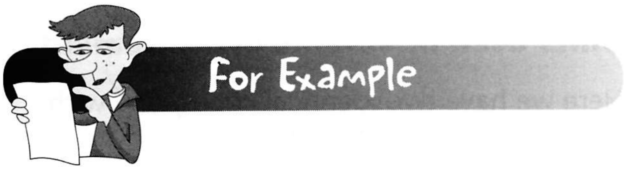
2 $25\,000 \div 1000 = 25$

$\therefore 25\,000\ mm^3 = 25\ cm^3$

Capacity

Capacity is a measure of the amount of liquid within a container. The basic unit we use is the litre (L):

1000 millilitres (mL) = 1 litre (L)
1000 litres (L) = 1 kilolitre (kL)
1000 kilolitres (kL) = 1 megalitre (ML)



1 Convert:

a 3.2 litres = ____ mL

b 7600 mL = ____ L

c 4.28 kL = ____ L

d 2742 L = ____ kL

e 7 ML = ____ mL

2 Jenz opened a 2-litre bottle of cordial and poured 220 mL into each of four glasses. How much remains in the bottle (in litres)?

1 a $3.2 \times 1000 = 3200$

$\therefore$ 3200 mL

b $7600 \div 1000 = 7.6$

$\therefore$ 7.6 L

c $4.28 \times 1000 = 4280$

$\therefore$ 4280 L

d $2742 \div 1000 = 2.742$

$\therefore$ 2.742 kL

e $7 \times 1000 \times 1000 \times 1000$

$= 7\,000\,000\,000$

$\therefore$ 7 000 000 000 mL

2 Remainder in bottle $= 2000 - 4 \times 220$

$= 2000 - 880$

$= 1120$

$\therefore$ 1120 mL

$\therefore$ 1.12 L remains

Capacity and Volume

$1\ cm^3 = 1\ mL$

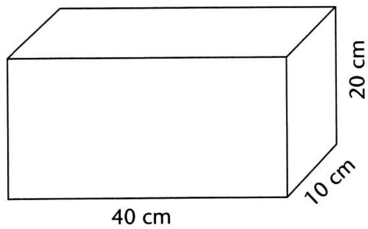
$\therefore$ $1000\ cm^3 = 1\ L$

Also, $cm^3 = cc$ [cc = cubic centimetres]

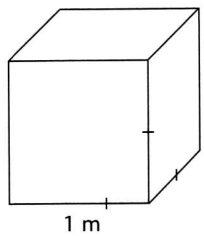
$\therefore$ $1000\ cc = 1\ L$

Find the capacity, in litres, of these containers:

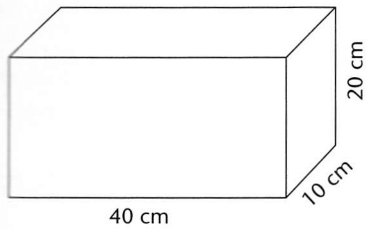
1



2



1



$V = lbh$

$= 40 \times 10 \times 20$

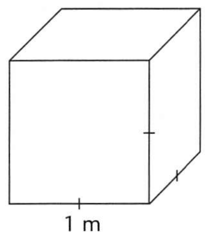
$= 8000$

$\therefore$ Volume is 8000 cm^3

$\therefore$ Capacity = 8000 ÷ 1000 = 8

Capacity is 8 litres.

2 1 m = 100 cm



$V = lbh$

$\therefore V = 100 \times 100 \times 100$

$= 1\,000\,000$

$\therefore$ Volume is 1 000 000 cm^3

$\therefore$ Capacity = 1 000 000 ÷ 1000

= 1000 L

= 1 kL

$\therefore$ Capacity is 1 kL. [i.e. 1 m^3 = 1 kL]